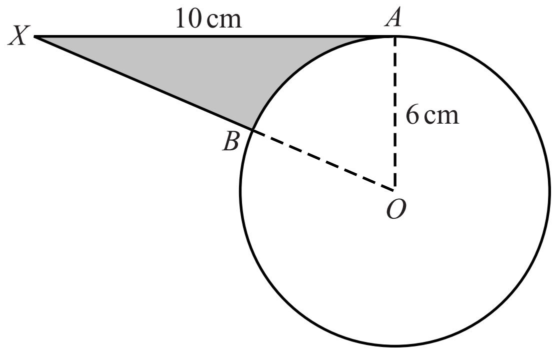
5 **(i)** Sketch, on the same diagram and for $0 \leqslant x \leqslant 2\pi$, the graphs of $y = \frac{1}{4} + \sin x$ and $y = \frac{1}{2}\cos 2x$. [4]

(ii) The x-coordinates of the points of intersection of the two graphs referred to in part **(i)** satisfy the equation $2\cos 2x - k\sin x = 1$. Find the value of k. [2]

7



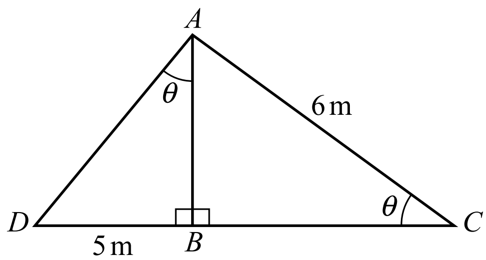
The diagram shows a circle, centre O and radius 6 cm. The tangent from X touches the circle at A and $XA = 10$ cm. The line from X to O cuts the circle at B.

(i) Show that angle AOB is approximately 1.03 radians. [1]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

8



In the diagram, angle ABC = angle $ABD = 90°$, $AC = 6$ m, $BD = 5$ m and angle ACB = angle $DAB = \theta$.

(i) Use each of the triangles ABC and ABD to express AB in terms of θ. [2]

(ii) Hence evaluate θ. [5]

7 The function f is defined, for $0° \leqslant x \leqslant 360°$, by $f(x) = 4 - \cos 2x$.

(i) State the amplitude and period of f. [2]

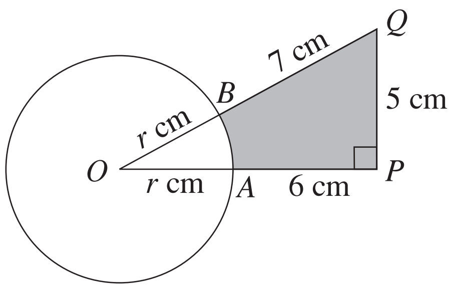
(ii) Sketch the graph of f, stating the coordinates of the maximum points. [4]

8 **(a)** Find all the angles between 0° and 360° which satisfy the equation

$$3(\sin x - \cos x) = 2(\sin x + \cos x).$$ [4]

(b) Find all the angles between 0 and 3 radians which satisfy the equation

$$1 + 3\cos^2 y = 4\sin y.$$ [4]



The diagram shows a right-angled triangle OPQ and a circle, centre O and radius r cm, which cuts OP and OQ at A and B respectively. Given that $AP = 6$ cm, $PQ = 5$ cm, $QB = 7$ cm and angle $OPQ = 90°$, find

(i) the length of the arc AB, [6]

(ii) the area of the shaded region. [4]

4 The function f is defined, for $0° \leqslant x \leqslant 360°$, by

$$\mathrm{f}(x) = a \sin(bx) + c,$$

where a, b and c are positive integers. Given that the amplitude of f is 2 and the period of f is 120°,

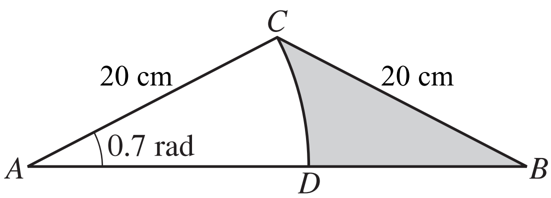
(i) state the value of a and of b. [2]

Given further that the minimum value of f is -1,

(ii) state the value of c, [1]

(iii) sketch the graph of f. [3]

10



The diagram shows an isosceles triangle ABC in which $BC = AC = 20$ cm, and angle $BAC = 0.7$ radians. DC is an arc of a circle, centre A. Find, correct to 1 decimal place,

(i) the area of the shaded region, [4]

(ii) the perimeter of the shaded region. [4]

10 **(a)** Solve, for $0° < x < 360°$,

$$4\tan^2 x + 15\sec x = 0.$$ [4]

(b) Given that $y > 3$, find the smallest value of y such that

$$\tan(3y - 2) = -5.$$ [4]

3 Given that θ is acute and that $\sin\theta = \dfrac{1}{\sqrt{3}}$, express, without using a calculator, $\dfrac{\sin\theta}{\cos\theta - \sin\theta}$ in the form $a + \sqrt{b}$, where a and b are integers. [5]

5 The function f is defined, for $0° \leqslant x \leqslant 180°$, by

$$f(x) = A + 5\cos Bx,$$

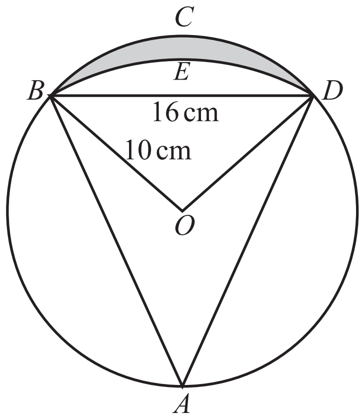
where A and B are constants.

(i) Given that the maximum value of f is 3, state the value of A. [1]

(ii) State the amplitude of f. [1]

(iii) Given that the period of f is 120°, state the value of B. [1]

(iv) Sketch the graph of f. [3]



The diagram, which is not drawn to scale, shows a circle $ABCDA$, centre O and radius 10 cm. The chord BD is 16 cm long. BED is an arc of a circle, centre A.

(i) Show that the length of AB is approximately 17.9 cm.

For the shaded region enclosed by the arcs BCD and BED, find

(ii) its perimeter, **(iii)** its area.

[11]

8 **(a)** Solve, for $0 \leqslant x \leqslant 2$, the equation $1 + 5\cos 3x = 0$, giving your answer in radians correct to 2 decimal places. [3]

(b) Find all the angles between 0° and 360° such that

$$\sec y + 5\tan y = 3\cos y.$$ [5]

10 The function f is defined, for $0° \leqslant x \leqslant 180°$, by

$$f(x) = 3\cos 4x - 1.$$

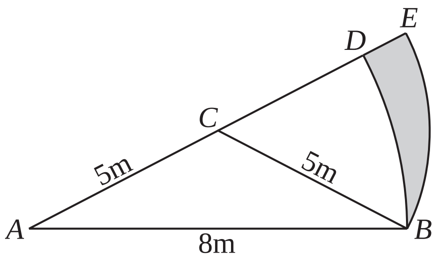
(i) Solve the equation $f(x) = 0$. [3]

(ii) State the amplitude of f. [1]

(iii) State the period of f. [1]

(iv) State the maximum and minimum values of f. [2]

(v) Sketch the graph of $y = f(x)$. [3]



The diagram shows an isosceles triangle *ABC* in which $AB = 8$ m, $BC = CA = 5$ m. *ABDA* is a sector of the circle, centre *A* and radius 8 m. *CBEC* is a sector of the circle, centre *C* and radius 5 m.

(i) Show that angle *BCE* is 1.287 radians correct to 3 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [4]

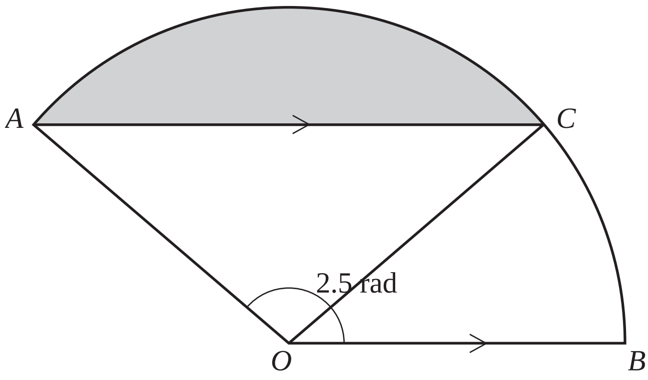
10 **(a)** Given that $a = \sec x + \operatorname{cosec} x$ and $b = \sec x - \operatorname{cosec} x$, show that

$$a^2 + b^2 \equiv 2\sec^2 x \operatorname{cosec}^2 x.$$ [4]

(b) Find, correct to 2 decimal places, the values of y between 0 and 6 radians which satisfy the equation

$$2\cot y = 3\sin y.$$ [5]

11



The diagram shows a sector $OACB$ of a circle, centre O, in which angle AOB = 2.5 radians. The line AC is parallel to OB.

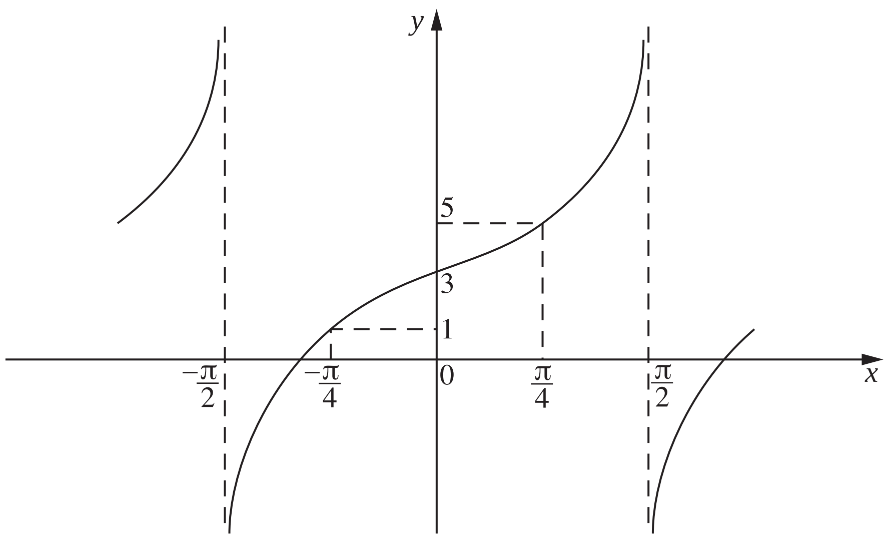
(i) Show that angle $AOC = (5 - \pi)$ radians. [3]

Given that the radius of the circle is 12 cm, find

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [3]

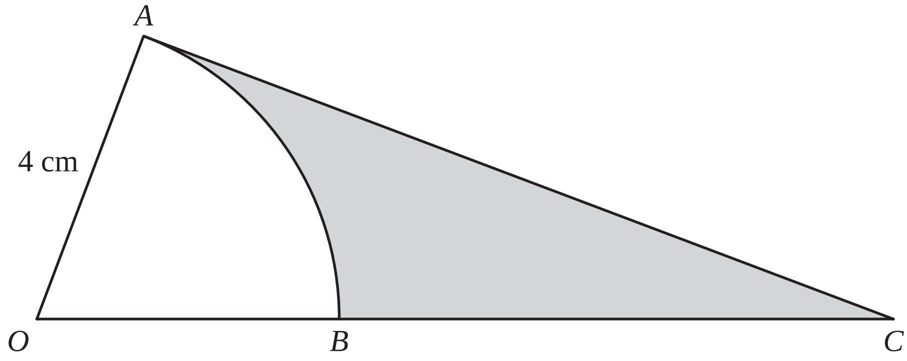
2



The diagram shows part of the graph of $y = a \tan (bx) + c$. Find the value of

(i) c, **(ii)** b, **(iii)** a. [3]

7



The diagram shows a sector OAB of a circle, centre O, radius 4 cm. The tangent to the circle at A meets the line OB extended at C. Given that the area of the sector OAB is $10\,\text{cm}^2$, calculate

(i) the angle AOB in radians, [2]

(ii) the perimeter of the shaded region. [4]

11 Solve the equation

(i) $3\sin x + 5\cos x = 0$ for $0° < x < 360°$, [3]

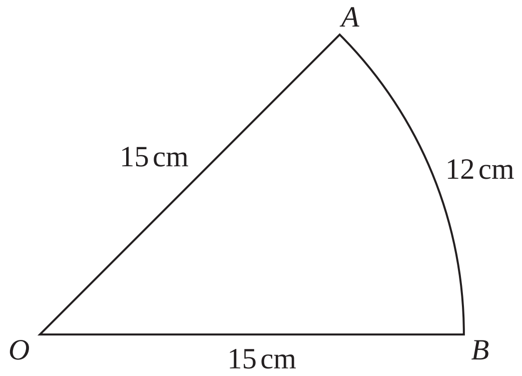
(ii) $3\tan^2 y - \sec y - 1 = 0$ for $0° < y < 360°$, [5]

(iii) $\sin(2z - 0.6) = 0.8$ for $0 < z < 3$ radians. [4]

3 Show that $\dfrac{1-\cos^2\theta}{\sec^2\theta - 1} = 1 - \sin^2\theta$. [4]

6 **(a)** Given that $\sin x = p$, find an expression, in terms of p, for $\sec^2 x$. [2]

(b) Prove that $\sec A \operatorname{cosec} A - \cot A \equiv \tan A$. [4]



The diagram shows a sector *AOB* of a circle, centre *O*, radius 15 cm. The length of the arc *AB* is 12 cm.

(i) Find, in radians, angle *AOB*. [2]

(ii) Find the area of the sector *AOB*. [2]

11 **(a)** Find all the angles between 0º and 360º which satisfy

(i) $2\sin x - 3\cos x = 0$, [3]

(ii) $2\sin^2 y - 3\cos y = 0$. [5]

(b) Given that $0 \leqslant z \leqslant 3$ radians, find, correct to 2 decimal places, all the values of z for which $\sin(2z + 1) = 0.9$. [3]

10 Solve

(i) $4\sin x = \cos x$ for $0° < x < 360°$, [3]

(ii) $3 + \sin y = 3\cos^2 y$ for $0° < y < 360°$, [5]

(iii) $\sec\left(\frac{z}{3}\right) = 4$ for $0 < z < 5$ radians. [3]

6 **(a)** **(i)** On the same diagram, sketch the curves $y = \cos x$ and $y = 1 + \cos 2x$ for $0 \leqslant x \leqslant 2\pi$. [3]

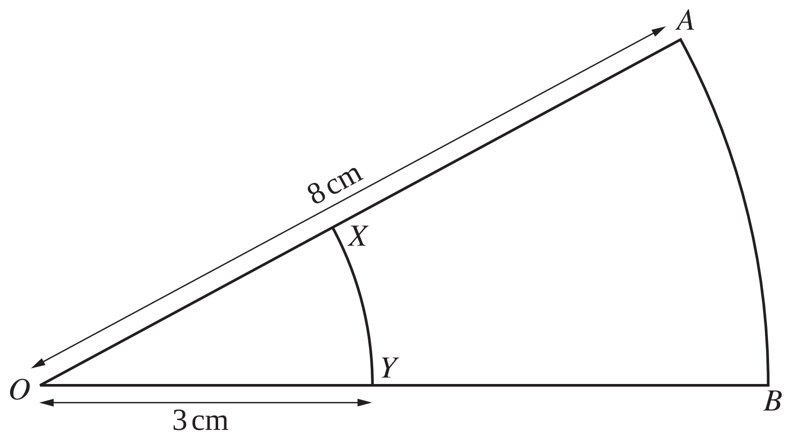
(ii) Hence state the **number** of solutions of the equation

$$\cos 2x - \cos x + 1 = 0 \text{ where } 0 \leqslant x \leqslant 2\pi.$$ [1]

(b) The function f is given by $f(x) = 5\sin 3x$. Find

(i) the amplitude of f, [1]

(ii) the period of f. [1]



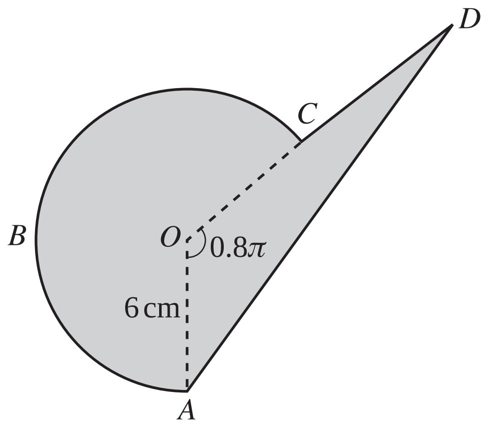
The diagram shows a sector OXY of a circle centre O, radius 3 cm and a sector OAB of a circle centre O, radius 8 cm. The point X lies on the line OA and the point Y lies on the line OB. The perimeter of the region $XABYX$ is 15.5 cm. Find

(i) the angle AOB in radians, [3]

(ii) the ratio of the area of the sector OXY to the area of the region $XABYX$ in the form $p : q$, where p and q are integers. [4]

4 **(a)** Given that $\sin x = p$ and $\cos x = 2p$, where x is acute, find the exact value of p and the exact value of $\operatorname{cosec} x$. [3]

(b) Prove that $(\cot x + \tan x)(\cot x - \tan x) = \dfrac{1}{\sin^2 x} - \dfrac{1}{\cos^2 x}$. [3]



The diagram represents a company logo *ABCDA*, consisting of a sector *OABCO* of a circle, centre *O* and radius 6 cm, and a triangle *AOD*. Angle $AOC = 0.8\pi$ radians and *C* is the mid-point of *OD*. Find

(i) the perimeter of the logo, [7]

(ii) the area of the logo. [5]

11 **(a)** Solve, for $0 < x < 3$ radians, the equation $4\sin x - 3 = 0$, giving your answers correct to 2 decimal places. [3]

(b) Solve, for $0° < y < 360°$, the equation $4\operatorname{cosec} y = 6\sin y + \cot y$. [6]

where n is a positive integer and $\binom{}{r} = \dfrac{n!}{(n-r)!r!}$.