(a) Describe fully a single transformation which maps both

(i) $A$ onto $C$ and $B$ onto $D$, 

(ii) $A$ onto $D$ and $B$ onto $C$, 

(iii) $A$ onto $P$ and $B$ onto $Q$. 

(b) Describe fully a single transformation which maps triangle $0AB$ onto triangle $JFE$. 

(c) The matrix $M$ is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

(i) Describe the transformation which $M$ represents. 

(ii) Write down the co-ordinates of $P$ after transformation by matrix $M$. 

(d) (i) Write down the matrix $R$ which represents a rotation by $90^\circ$ anticlockwise about $0$. 

(ii) Write down the letter representing the new position of $F$ after the transformation $RM(F)$. 


(a) Describe fully the single transformation which maps

(i) shape \( A \) onto shape \( B \),

(ii) shape \( B \) onto shape \( C \),

(iii) shape \( A \) onto shape \( D \),

(iv) shape \( B \) onto shape \( E \),

(v) shape \( B \) onto shape \( F \),

(vi) shape \( A \) onto shape \( G \).

(b) A transformation is represented by the matrix \( \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \).

Which shape above is the image of shape \( A \) after this transformation?  

(c) Find the 2 by 2 matrix representing the transformation which maps

(i) shape \( B \) onto shape \( D \),

(ii) shape \( A \) onto shape \( G \).
(a) Describe fully the single transformation which maps
   (i) triangle $X$ onto triangle $P$,  
   (ii) triangle $X$ onto triangle $Q$,  
   (iii) triangle $X$ onto triangle $R$,  
   (iv) triangle $X$ onto triangle $S$.  

(b) Find the 2 by 2 matrix which represents the transformation that maps
   (i) triangle $X$ onto triangle $Q$,  
   (ii) triangle $X$ onto triangle $S$.  

Transformation $M$ is reflection in the line $y = x$.

(a) The point $A$ has co-ordinates $(2, 1)$.

Find the co-ordinates of

(i) $T(A)$, \[1\]

(ii) $MT(A)$. \[2\]

(b) Find the 2 by 2 matrix $M$, which represents the transformation $M$. \[2\]

(c) Show that, for any value of $k$, the point $Q (k - 2, k - 3)$ maps onto a point on the line $y = x$ following the transformation $TM(Q)$. \[3\]

(d) Find $M^{-1}$, the inverse of the matrix $M$. \[2\]

(e) $N$ is the matrix such that $N + \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 0 & 0 \end{pmatrix}$.

(i) Write down the matrix $N$. \[2\]

(ii) Describe completely the single transformation represented by $N$. \[3\]
Answer the whole of this question on a sheet of graph paper.

(a) Draw and label $x$ and $y$ axes from $-6$ to $6$, using a scale of $1$ cm to $1$ unit. \[1\]

(b) Draw triangle $ABC$ with $A(2,1)$, $B(3,3)$ and $C(5,1)$. \[1\]

(c) Draw the reflection of triangle $ABC$ in the line $y = x$. Label this $A_1B_1C_1$. \[2\]

(d) Rotate triangle $A_1B_1C_1$ about $(0,0)$ through $90^\circ$ anti-clockwise. Label this $A_2B_2C_2$. \[2\]

(e) Describe fully the single transformation which maps triangle $ABC$ onto triangle $A_2B_2C_2$. \[2\]

(f) A transformation is represented by the matrix \[
\begin{pmatrix}
1 & 0 \\
-1 & 1
\end{pmatrix}
\]

(i) Draw the image of triangle $ABC$ under this transformation. Label this $A_3B_3C_3$. \[3\]

(ii) Describe fully the single transformation represented by the matrix \[
\begin{pmatrix}
1 & 0 \\
-1 & 1
\end{pmatrix}
\] \[2\]

(iii) Find the matrix which represents the transformation that maps triangle $A_3B_3C_3$ onto triangle $ABC$. \[2\]
(a) On the grid, draw the enlargement of the triangle $T$, centre $(0, 0)$, scale factor $\frac{1}{2}$. \[2\]
(b) The matrix \( \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \) represents a transformation.

(i) Calculate the matrix product \( \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 8 & 8 & 2 \\ 4 & 8 & 8 \end{pmatrix} \).

Answer(b)(i) [2]

(ii) On the grid, draw the image of the triangle \( T \) under this transformation.

(iii) Describe fully this single transformation.

Answer(b)(iii) [2]

(c) Describe fully the single transformation which maps

(i) triangle \( T \) onto triangle \( P \),

Answer(c)(i) [2]

(ii) triangle \( T \) onto triangle \( Q \).

Answer(c)(ii) [3]

(d) Find the 2 by 2 matrix which represents the transformation in part (c)(ii).

Answer(d) \( \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \) [2]
(a) On the grid, draw

(i) the translation of triangle $T$ by the vector $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$, \[2\]

(ii) the rotation of triangle $T$ about $(0, 0)$, through $90^\circ$ clockwise. \[2\]

(b) Describe fully the single transformation that maps

(i) triangle $T$ onto triangle $U$,

Answer(b)(i) \[2\]

(ii) triangle $T$ onto triangle $V$. 

Answer(b)(ii) \[3\]
(a) Draw the reflection of triangle $T$ in the line $y = 6$.

Label the image $A$. \[2\]

(b) Draw the translation of triangle $T$ by the vector $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$.

Label the image $B$. \[2\]
Answer the whole of this question on a sheet of graph paper.

(a) Using a scale of 1 cm to represent 1 unit on each axis, draw an x-axis for \(-6 \leq x \leq 10\) and a y-axis for \(-8 \leq y \leq 8\).
Copy the word EXAM onto your grid so that it is exactly as it is in the diagram above.
Mark the point \(P(6,6)\). [2]

(b) Draw accurately the following transformations.
(i) Reflect the letter \(E\) in the line \(x = 0\). [2]
(ii) Enlarge the letter \(X\) by scale factor 3 about centre \(P(6,6)\). [2]
(iii) Rotate the letter \(A\) 90° anticlockwise about the origin. [2]
(iv) Stretch the letter \(M\) vertically with scale factor 2 and x-axis invariant. [2]

(c) (i) Mark and label the point \(Q\) so that \(\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}\). [1]
(ii) Calculate \(|\overrightarrow{PQ}|\) correct to two decimal places. [2]
(iii) Mark and label the point \(S\) so that \(\overrightarrow{PS} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}\). [1]
(iv) Mark and label the point \(R\) so that \(PQRS\) is a parallelogram. [1]
Use one of the letters $A, B, C, D, E$ or $F$ to answer the following questions.

(i) Which triangle is $T$ mapped onto by a **translation**? Write down the translation vector. [2]

(ii) Which triangle is $T$ mapped onto by a **reflection**? Write down the equation of the mirror line. [2]

(iii) Which triangle is $T$ mapped onto by a **rotation**? Write down the coordinates of the centre of rotation. [2]

(iv) Which triangle is $T$ mapped onto by a **stretch** with the $x$-axis invariant? Write down the scale factor of the stretch. [2]

(v) $M = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$. Which triangle is $T$ mapped onto by $M$?

Write down the name of this transformation. [2]

(b) $P = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}$, $Q = (-1 \quad -2)$, $R = (1 \quad 2 \quad 3)$, $S = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$.

Only some of the following matrix operations are possible with matrices $P, Q, R$ and $S$ above. $\text{PQ}, \quad \text{QP}, \quad \text{P + Q}, \quad \text{PR}, \quad \text{RS}$

Write down and calculate each matrix operation that is possible. [6]
4 Answer the whole of this question on a sheet of graph paper.

(a) Draw $x$- and $y$-axes from $-8$ to $8$ using a scale of $1$ cm to $1$ unit.
    Draw triangle $ABC$ with $A(2, 2)$, $B(5, 2)$ and $C(5, 4)$. [2]

(b) Draw the image of triangle $ABC$ under a translation of \[ \begin{pmatrix} -9 \\ 3 \end{pmatrix} \].
    Label it $A_1B_1C_1$. [2]

(c) Draw the image of triangle $ABC$ under a reflection in the line $y = -1$.
    Label it $A_2B_2C_2$. [2]

(d) Draw the image of triangle $ABC$ under an enlargement, scale factor $2$, centre $(6,0)$.
    Label it $A_3B_3C_3$. [2]

(e) The matrix \[ \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \] represents a transformation.
    (i) Draw the image of triangle $ABC$ under this transformation. Label it $A_4B_4C_4$. [2]
    (ii) Describe fully this single transformation. [2]

(f) (i) Draw the image of triangle $ABC$ under a stretch, factor $1.5$, with the $y$-axis invariant.
    Label it $A_5B_5C_5$. [2]
    (ii) Find the $2$ by $2$ matrix which represents this transformation. [2]

7 Answer the whole of this question on a sheet of graph paper.

(a) Draw $x$ and $y$ axes from $0$ to $12$ using a scale of $1$ cm to $1$ unit on each axis. [1]

(b) Draw and label triangle $T$ with vertices $(8, 6)$, $(6, 10)$ and $(10, 12)$. [1]

(c) Triangle $T$ is reflected in the line $y = x$.
    (i) Draw the image of triangle $T$. Label this image $P$. [2]
    (ii) Write down the matrix which represents this reflection. [2]

(d) A transformation is represented by the matrix \[ \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \]
    (i) Draw the image of triangle $T$ under this transformation. Label this image $Q$. [2]
    (ii) Describe fully this single transformation. [3]

(e) Triangle $T$ is stretched with the $y$-axis invariant and a stretch factor of $1/2$.
    Draw the image of triangle $T$. Label this image $R$. [2]
The diagram shows triangles $P$, $Q$, $R$, $S$, $T$, and $U$.

(a) Describe fully the single transformation which maps triangle

(i) $T$ onto $P$, [2]
(ii) $Q$ onto $T$, [2]
(iii) $T$ onto $R$, [2]
(iv) $T$ onto $S$, [3]
(v) $U$ onto $Q$. [3]

(b) Find the 2 by 2 matrix representing the transformation which maps triangle

(i) $T$ onto $R$, [2]
(ii) $U$ onto $Q$. [2]
(a) Describe fully the single transformation which maps

(i) triangle $T$ onto triangle $U$,

Answer(a)(i) ............................................................................................................. [2]

(ii) triangle $T$ onto triangle $V$,

Answer(a)(ii) ............................................................................................................. [3]
(iii) triangle $T$ onto triangle $W$.

Answer(a)(iii) ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~[3]

(iv) triangle $U$ onto triangle $X$.

Answer(a)(iv) ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~[3]

(b) Find the matrix representing the transformation which maps

(i) triangle $U$ onto triangle $V$,

Answer(b)(i) ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~[2]

(ii) triangle $U$ onto triangle $X$.

Answer(b)(ii) ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~[2]
2 (a)

(i) Draw the image when triangle \(A\) is reflected in the line \(y = 0\).
Label the image \(B\). [2]

(ii) Draw the image when triangle \(A\) is rotated through \(90^\circ\) anticlockwise about the origin.
Label the image \(C\). [2]

(iii) Describe fully the single transformation which maps triangle \(B\) onto triangle \(C\).

Answer (a)(iii) .............................................................................................................................................. [2]

(b) Rotation through \(90^\circ\) anticlockwise about the origin is represented by the matrix \(M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\).

(i) Find \(M^{-1}\), the inverse of matrix \(M\).

\[
Answer(b)(i) \quad M^{-1} = \begin{pmatrix} \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} \end{pmatrix}
\]

[2]

(ii) Describe fully the single transformation represented by the matrix \(M^{-1}\).

Answer (b)(ii) .............................................................................................................................................. [2]
Draw the images of the following transformations on the grid above.

(i) Translation of triangle $A$ by the vector $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$. Label the image $B$. [2]

(ii) Reflection of triangle $A$ in the line $x = 3$. Label the image $C$. [2]

(iii) Rotation of triangle $A$ through $90^\circ$ anticlockwise around the point $(0, 0)$. Label the image $D$. [2]

(iv) Enlargement of triangle $A$ by scale factor $-4$, with centre $(0, 1)$. Label the image $E$. [2]
where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$. 