

stream with the bank, find

- (a) the distance where he lands from the point directly opposite A,
 (b) the time taken to cross the river.

- 13 An aeroplane can fly at 300 km h^{-1} in still air and there is a wind blowing from the direction 060° at a constant rate of 40 km h^{-1} .
- (a) What course should the pilot take to reach an airfield 200 km due N of his starting point?
 (b) If he takes this course, how long will the flight last?
 (c) What course should be taken for the return flight (the wind being as before) and how long will this flight last?
- 14 The speed of a helicopter in still air is $v \text{ km h}^{-1}$. The pilot leaves A and flies on a course 067° . There is a wind of 50 km h^{-1} blowing from the direction 020° . After 45 minutes, the helicopter is above a point B which is due E of A. Find
- (a) the value of v to the nearest km h^{-1} ,
 (b) the distance AB.

RELATIVE VELOCITY

Consider two aeroplanes A and B flying at the same height. A is flying at 200 km h^{-1} due N and B at 400 km h^{-1} on a course 030° . These are their velocities **relative to the ground** and are their **true** velocities. To the pilot of A, however, B will seem to have a different velocity because he himself is moving. We call this apparent velocity, the **velocity of B relative to A** and we write this as \vec{BvA} . Similarly B will see A moving with the velocity of A relative to B or \vec{AvB} .

In general, suppose \mathbf{a} represents the velocity of A and \mathbf{b} the velocity of B (Fig. 21.13). Then $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$.

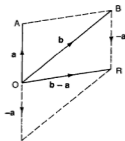


Fig. 21.13

Let us reduce A to rest (theoretically) by introducing a velocity $-\mathbf{a}$. A is now not moving. In order to preserve their relative position, we must also give B the same velocity $-\mathbf{a}$. B now has 2 velocities, \mathbf{b} and $-\mathbf{a}$ and the resultant of these is \vec{OR} . \vec{OR} represents the velocity of B as seen from A (as A is not moving). But $\vec{OR} = \vec{AB} = \mathbf{b} - \mathbf{a}$. Hence the velocity of B relative to A is $\mathbf{b} - \mathbf{a}$ or $\vec{BvA} = \mathbf{b} - \mathbf{a}$. Similarly, $\vec{AvB} = \mathbf{a} - \mathbf{b}$.

We can now find \vec{BvA} for the two aeroplane. From Fig. 21.14,

$$|\vec{BvA}|^2 = 200^2 + 400^2 - 2 \times 200 \times 400 \times \cos 30^\circ$$

giving $|\vec{BvA}| = 248$.

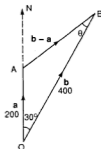


Fig. 21.14

Also $\frac{\sin \theta}{200} = \frac{\sin 30^\circ}{248}$ giving $\theta = 23.8^\circ$.

Therefore $\angle NAB = 30^\circ + 23.8^\circ = 53.8^\circ$.

Hence the velocity of B relative to A is 248 km h^{-1} in the direction $N53.8^\circ E$.

Note: Problems on relative velocity may also be solved by drawing if the rubric of the question does not forbid it (see Example 6).

[We can also work with unit vectors as follows:

Take i along \vec{OE} and j along \vec{ON} (Fig. 21.15)

$$a = 200j$$

$$\text{and } b = (400 \sin 30^\circ)i + (400 \cos 30^\circ)j \\ = 200i + (200\sqrt{3})j$$

$$\text{So } \vec{BvA} = b - a \\ = 200i + (200\sqrt{3} - 200)j \\ = 200i + 146.4j$$

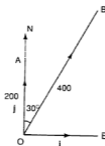


Fig. 21.15

Hence $|\vec{BvA}|^2 = 200^2 + 146.4^2$ (Fig. 21.16)

giving $|\vec{BvA}|^2 = 248$

and $\tan \phi = \frac{200}{146.4}$ giving $\phi \approx 53.8^\circ$.

Hence \vec{BvA} is 248 km h^{-1} in the direction $N53.8^\circ E$ as before.]

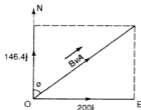


Fig. 21.16

Example 4

Rain is falling vertically at 5 km h^{-1} . A man is sitting by the window of a train travelling at 40 km h^{-1} . In what direction do the raindrops appear to cross the windows of the train?

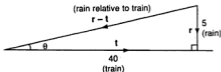


Fig. 21.17

The vectors are shown in Fig. 21.17.

The velocity of the rain relative to the train = $r - t$.

So $\tan \theta = \frac{5}{40}$ giving $\theta = 7.1^\circ$.

Thus the rain drops appear to cross the windows at 7.1° to the horizontal.

Example 5

A ship is sailing due north at a constant speed of 12 knots. A destroyer sailing at 36 knots is 30 nautical miles due east of the ship. At this moment, the destroyer is ordered to intercept the ship. Find

- the course which the destroyer should take,
- the velocity of the destroyer relative to the ship,
- the time taken for the destroyer to reach the ship.

[It is assumed that both the ship and the destroyer do not change their velocities.]

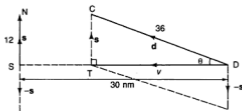


Fig. 21.18

Fig. 21.18 shows the positions of the ship S and the destroyer D. We reduce S to rest by introducing a velocity of 12 knots due south to both S and D. Then the course of D is \vec{DC} and, to intercept S, its track (\vec{DT}) must lie along \vec{DS} .

(a) From $\triangle DTC$, $\sin \theta = \frac{12}{36}$ giving $\theta = 19.5^\circ$.

Hence the course the destroyer should take is $270^\circ + 19.5^\circ = 289.5^\circ$.

(b) The velocity of D relative to S is

$$\vec{DvS} = \vec{DT}$$

Therefore $|\vec{DvS}|^2 = 36^2 - 12^2$ giving $|\vec{DvS}| = 33.9$.

Hence \vec{DvS} is 33.9 knots due W.

(c) Time taken = $\frac{30}{33.9}$ h = 53 min.

Hence the destroyer will intercept the ship in 53 minutes.

[By vectors, take \mathbf{i} along DT and \mathbf{j} along TC, then

$$\vec{DvS} = \mathbf{d} - \mathbf{s}$$

$$\begin{aligned} \text{i.e. } v\mathbf{i} &= (36 \cos \theta)\mathbf{i} + (36 \sin \theta)\mathbf{j} - 12\mathbf{j} \\ &= (36 \cos \theta)\mathbf{i} + (36 \sin \theta - 12)\mathbf{j} \end{aligned}$$

Therefore $0 = 36 \sin \theta - 12$ (equating the \mathbf{j} components)

$$\text{i.e. } \sin \theta = \frac{12}{36} \text{ giving } \theta = 19.5^\circ \text{ as before.}$$

$$\text{Also } v = 36 \cos \theta = 33.9.$$

Hence \vec{DvS} is 33.9 knots due W and the time taken can be calculated as before.

Example 6

To a man travelling in a car in the direction 060° at 40 km h^{-1} , the wind appears to be blowing from the east at 10 km h^{-1} . Find, by drawing or calculation, the true velocity of the wind.

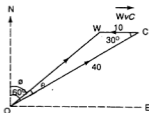


Fig. 21.19.

1 By calculation (using trigonometry)

Fig. 21.19 shows the vectors: \vec{OC} = velocity of car, \vec{OW} = true velocity of wind and $\vec{CW} = \vec{WvC}$ (the velocity of the wind relative to the car).

$$\text{Since } \vec{WvC} = \vec{OW} - \vec{OC}$$

$$\vec{OW} = \vec{WvC} + \vec{OC}$$

By the cosine rule,

$$|\vec{OW}|^2 = 40^2 + 10^2 - 2 \times 40 \times 10 \times \cos 30^\circ \text{ giving } |\vec{OW}| \approx 31.7.$$

$$\text{Also, } \frac{\sin \theta}{10} = \frac{\sin 30^\circ}{31.7} \text{ giving } \theta \approx 9.1^\circ.$$

Hence the true velocity of the wind is 31.7 km h^{-1} towards the direction $(60^\circ - 9.1^\circ) = 50.9^\circ$ or from the direction 230.9° .

2 By calculation (using vectors)

[By vectors, take \mathbf{i} along \vec{OE} and \mathbf{j} along \vec{ON} .

$$\vec{OC} = (40 \sin 60^\circ)\mathbf{i} + (40 \cos 60^\circ)\mathbf{j} = (20\sqrt{3})\mathbf{i} + 20\mathbf{j}$$

$$\vec{WvC} = -10\mathbf{i}$$

$$\vec{OW} = \vec{OC} + \vec{WvC} = (20\sqrt{3} - 10)\mathbf{i} + 20\mathbf{j}$$

$$|\vec{OW}|^2 = (20\sqrt{3} - 10)^2 + 20^2 \text{ giving } |\vec{OW}| \approx 31.7.$$

$$\text{Also, } \tan \phi = \frac{20\sqrt{3} - 10}{20} \text{ giving } \phi \approx 50.9^\circ.$$

Thus we obtain the same results as before.

- 3 First draw a sketch and label it with all the information given (it should be a rough version of Fig. 21.19). The actual drawing must be done carefully. Choose a suitable scale to ensure reasonably accurate results, say 1 cm for 4 km h^{-1} .

Draw a north line ON as in Fig. 21.19.

From O, draw OC 10 cm long with $\angle NOC = 60^\circ$.

From C, draw C'W 2.5 cm long parallel to OE.

Join OW.

Measure OW (and convert to km h^{-1}) and $\angle NOW$.

Compare with the calculated values above.

Exercise 21.2 (Answers on page 649.)

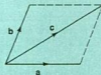
- 1 Aeroplane A is flying due N at 150 km h^{-1} . Aeroplane B is flying due E at 200 km h^{-1} . Find the velocity of B relative to A.
- 2 Two cars A and B are travelling on roads which cross at right angles. Car A is travelling due east at 60 km h^{-1} , car B is travelling at 40 km h^{-1} due north, both going towards the crossing. Find the velocity of B relative to A. [The magnitude and direction must be given].
- 3 A passenger is on the deck of a ship sailing due east at 25 km h^{-1} . The wind is blowing from the north-east at 10 km h^{-1} . What is the velocity of the wind relative to the passenger?
- 4 A road (running north-south) crosses a railway line at right angles. A passenger in a car travelling north at 60 km h^{-1} and 600 m south of the bridge, sees a train, travelling west at 90 km h^{-1} , which is 800 m east of the bridge. Find the velocity of the train relative to the car.

- 5 To a man in a car travelling at 20 km h^{-1} north-east, the wind appears to blow from the west with speed 16 km h^{-1} . Find the true velocity of the wind.
- 6 Aeroplane A is flying at 400 km h^{-1} north-east and sees aeroplane B which is apparently flying north at 500 km h^{-1} . What is the true velocity of B?
- 7 An unidentified aircraft is reported as flying due north at 500 km h^{-1} . A fighter plane, which is 100 km on a bearing of 225° from the unknown plane is ordered to contact it. If the fighter can fly at 800 km h^{-1} , what course should it take? After how long will it be in contact?
- 8 A man on a ship steaming due south at 12 km h^{-1} sees a balloon apparently travelling due west at 15 km h^{-1} . Find the true velocity of the balloon.
- 9 Two ships A and B are 30 km apart with B due south of A. A is sailing at 10 km h^{-1} in the direction 120° while B is sailing at 15 km h^{-1} in the direction 045° .
- Find the velocity of A relative to B.
 - Calculate the time taken for B to be due west of A.
- 10 A helicopter is flying on a course 060° with speed 100 km h^{-1} . An aeroplane, which is 50 km due east of the helicopter, can fly at 200 km h^{-1} . What course should the aeroplane take to intercept the helicopter?
- 11 To a ship sailing due N at 20 km h^{-1} another ship appears to be moving with a velocity of 12 km h^{-1} in the direction 120° . Find the true velocity of this ship.
- 12 A ship A is heading north at 20 km h^{-1} and at 1200 h is 50 km south-east of a ship B. If B steers at 25 km h^{-1} so as to just intercept A, find
- the direction in which B must travel,
 - the time when the interception takes place.
- 13 A man is walking along a horizontal road at 1.2 m s^{-1} . The rain is coming towards him and appears to be falling with a speed of 4 m s^{-1} in the direction which makes an angle of 60° with the horizontal. Find the actual speed of the rain and the angle this speed makes with the horizontal.
- Find the speed and the angle with the horizontal which the rain would appear to make if he walked at the same speed in the opposite direction.

SUMMARY

- *Composition of velocities*

If a body has 2 velocities a and b , the resultant velocity c is given by $c = a + b$. If a and b are represented by the sides of a parallelogram, then c is represented by the diagonal as shown in the diagram.



● *Resolution of velocities*

It is useful to resolve a velocity \mathbf{r} into 2 perpendicular components, usually along the x - and y -axes. Unit coordinate vectors \mathbf{i} and \mathbf{j} are often used as shown in the diagram.

$$\text{Also } |\mathbf{r}| = r = \sqrt{p^2 + q^2},$$

$$p = r \cos \theta, \quad q = r \sin \theta \quad \text{and} \quad \tan \theta = \frac{q}{p}$$

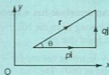
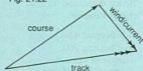


Fig. 21.21

● *Notation*

- course vector
- wind/current vector
- track vector

Fig. 21.22



The track vector is the resultant of the other 2 vectors.

$$\begin{array}{c} \text{→} \\ \text{(course)} \end{array} + \begin{array}{c} \text{→→} \\ \text{(wind/current)} \end{array} = \begin{array}{c} \text{→→→} \\ \text{(track)} \end{array}$$

● *Relative velocity*

\vec{BvA} means the velocity of B relative to A.

$$\vec{BvA} = \mathbf{b} - \mathbf{a}$$

where \mathbf{b} = velocity of B

and \mathbf{a} = velocity of A.

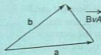


Fig. 21.23

REVISION EXERCISE 21 (Answers on page 649.)

- A river is 160 m wide and runs at 1.2 m s^{-1} between straight parallel banks. A man can row at 2 m s^{-1} in still water.
 - If he rows in a direction perpendicular to the banks, how far downstream will he land?
 - At what acute angle (to the nearest degree) to the bank should he now row to return to his starting point?
- A small aeroplane can fly at 200 km h^{-1} in still air. There is a wind of 50 km h^{-1} from the east. If the pilot wishes to fly due south, what course should he take and what is his ground speed?

The pilot keeps this course but the wind changes and the pilot finds that his ground speed is 200 km h^{-1} in the direction 190° . Calculate the new velocity of the wind.

- 3 An aeroplane can fly at 300 km h^{-1} relative to the air. If there is a wind of 60 km h^{-1} from the east, what is the least time in which the aeroplane can reach a point 600 km south-west of its starting point?
- 4 An aeroplane has a speed of $V \text{ km h}^{-1}$ in still air and there is a wind of $\frac{V}{4} \text{ km h}^{-1}$ blowing from the NE. Find the course that must be taken if the pilot wishes to reach a point due east.
- 5 A destroyer detects the presence of a vessel at a range of 30 nautical miles on a bearing of 060° . The vessel is steaming on a course of 150° at a speed of 15 knots. If the destroyer steams at 22 knots determine either by drawing or by calculation the course the destroyer must steer so that its velocity relative to the vessel is in a direction 060° . Hence determine the time taken for the destroyer to intercept the vessel if neither changes course. (C)
- 6 A helicopter whose speed in still air is 40 km h^{-1} , flies to an oil platform 60 km away on a bearing 060° . The wind velocity is 15 km h^{-1} from due north. Sketch a suitable triangle of velocities and find
 (i) the bearing on which the helicopter must fly,
 (ii) the time taken to reach the platform. (C)
- 7 Two canoeists, A and B, can paddle in still water at 6 m s^{-1} and 5 m s^{-1} respectively. They both set off at the same time from the same point on one bank of a river which has straight parallel banks, 240 m apart, and which flows at 3 m s^{-1} . A paddles in the direction that will take him across the river by the shortest distance whilst B paddles in the direction that will take him across the river in the shortest time. Determine
 (i) the direction in which A must paddle,
 (ii) the direction in which B must paddle,
 (iii) the time taken by each canoeist,
 (iv) the distance between the points at which they land. (C)
- 8 A plane flies in a straight line from London to Rome, a distance of 1400 km on a bearing of 135° . Given that the plane's speed in still air is 380 km h^{-1} , that the wind direction is 225° and that the journey takes 4 hours, determine
 (i) the wind speed,
 (ii) the direction the pilot should set for the flight.
 Find also the direction the pilot should set for the return flight, assuming that the speed and direction of the wind remain unchanged. (C)
- 9 A tanker is sailing on a fixed course due west at 30 km h^{-1} . At a time of 0900 a destroyer wishing to refuel is 160 km away on a bearing of 225° from the tanker. If the destroyer travels at 60 km h^{-1} , determine
 (i) the direction in which the destroyer should travel in order to reach the tanker,
 (ii) the time at which the destroyer reaches the tanker. (C)
- 10 A cyclist is travelling due north at 20 km h^{-1} and finds that the wind relative to him appears to be blowing from a direction 040° with a speed of 30 km h^{-1} . Find the true speed and direction of the wind.

Assuming that the speed and direction of the wind remain unchanged, find the speed of the wind relative to the cyclist when the cyclist is travelling due south at 20 km h^{-1} . (C)

- 11 An aircraft A is flying due east at 600 km h^{-1} and its bearing from aircraft B is 030° . If aircraft B has a speed of 1000 km h^{-1} find the direction in which B must fly in order to intercept A. If the aircraft are initially 50 km apart find the time taken, in minutes, for the interception to occur. (C)
- 12 An aircraft whose speed in still air is 300 km h^{-1} flies from A due north to B and back, where $AB = 500 \text{ km}$. The wind velocity is 120 km h^{-1} from 240° . Find
- the bearing on which the aircraft must fly for the outward journey,
 - the time of flight of the outward journey,
 - the bearing on which the aircraft must fly for the return journey. (C)
- 13 An aircraft is flying from a point A to a point B 400 km north-east of A, and a wind is blowing from the west at 50 km h^{-1} . The speed of the aircraft in still air is 300 km h^{-1} . Find
- the direction in which the aircraft must be headed,
 - the distance of the aircraft from B one hour after leaving A. (C)
- 14 (a) An aircraft flies in a straight line from a point A to a point B 200 km east of A. There is a wind blowing at 40 km h^{-1} from the direction 240° and the aircraft travels at 300 km h^{-1} in still air. Find
- the direction in which the pilot must steer the aircraft,
 - the time, to the nearest minute, for the journey from A to B.
- (b) To an observer in a ship sailing due north at 10 km h^{-1} , a second ship appears to be sailing due east at 24 km h^{-1} . Find the magnitude and direction of the actual velocity of the second ship. (C)
- 15 When a man walks at 4 km h^{-1} due west, the wind appears to blow from the south. When he walks at 8 km h^{-1} due west, the wind now appears to blow from the south-west. Find, by drawing or calculation, the true velocity of the wind.
- 16 Two ships A and B leave a harbour at the same time, A sailing due N at 20 km h^{-1} , B sailing at 25 km h^{-1} in the direction 060° . Calculate
- the velocity of B relative to A (stating the magnitude and direction),
 - the bearing of B from A in the subsequent motion,
 - the distance between the ships after 2 hours of sailing.
- 17 An aeroplane has a speed of $V \text{ km h}^{-1}$ in still air and flies in a straight line from A to B. There is a wind of speed $\frac{3}{4}V \text{ km h}^{-1}$ blowing from a direction making an angle θ with AB where $\sin \theta = \frac{4}{3}$. Find
- the angle which the course to be taken makes with AB,
 - the ground speed in terms of V .

- 18** A plane is scheduled to fly from London to Rome in $2\frac{1}{2}$ hours. Rome is 1400 km from London and on a bearing of 135° from London. Given that there is a wind blowing from the north at 120 km h^{-1} find, by calculation or drawing,
- the speed of the plane in still air,
 - the course which the pilot should set.
- Assuming that the velocity of the wind and the speed of the plane in still air remain unchanged, find
- the course which the pilot should set for the return journey.
 - the time taken, to the nearest minute. (C)
- 19** (a) An aircraft is flying due south at 350 km h^{-1} . The wind is blowing at 70 km h^{-1} from the direction θ° , where θ is acute. Given that the pilot is steering the aircraft in the direction 170° , find
- the value of θ ,
 - the speed of the aircraft in still air.
- (b) A man who swims at 1.2 m s^{-1} in still water wishes to cross a river which is flowing between straight parallel banks at 2 m s^{-1} . He aims downstream in a direction making an angle of 60° with the bank.
- Find
- the speed at which he travels,
 - the angle which his resultant velocity makes with the bank. (C)
- 20** (a) Two particles, A and B , are 60 m apart with B due west of A . Particle A is travelling at 9 m s^{-1} in a direction 300° and B is travelling at 12 m s^{-1} in a direction 030° . Find
- the magnitude and direction of the velocity of B relative to A ,
 - the time taken for B to be due north of A .
- (b) A wind is blowing from the direction 320° at 30 km h^{-1} . Find, by drawing or by calculation, the magnitude and direction of the velocity of the wind relative to a man who is cycling due east at 18 km h^{-1} .