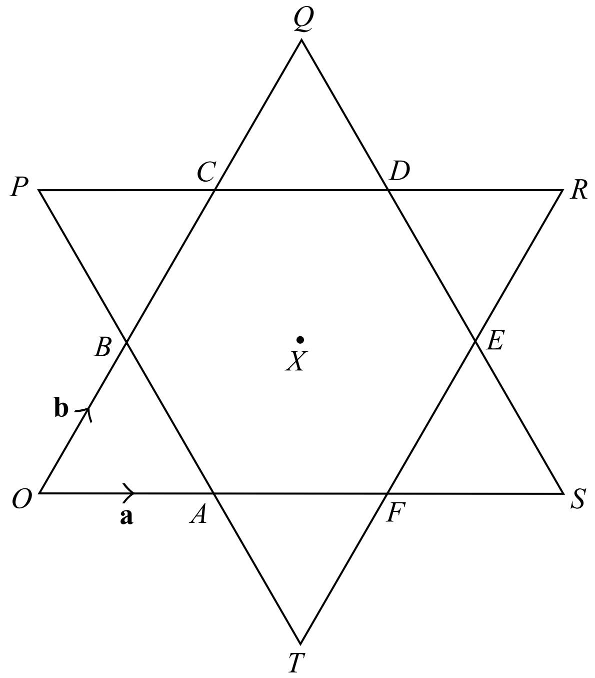
7



A star is made up of a regular hexagon, centre X, surrounded by 6 equilateral triangles.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Write the following vectors in terms of **a** and/or **b**, giving your answers in their simplest form.

(i) $\overrightarrow{OS}$, [1]

(ii) $\overrightarrow{AB}$, [1]

(iii) $\overrightarrow{CD}$, [1]

(iv) $\overrightarrow{OR}$, [2]

(v) $\overrightarrow{CF}$. [2]

(b) When $|\mathbf{a}| = 5$, write down the value of

(i) $|\mathbf{b}|$, [1]

(ii) $|\mathbf{a} - \mathbf{b}|$. [1]

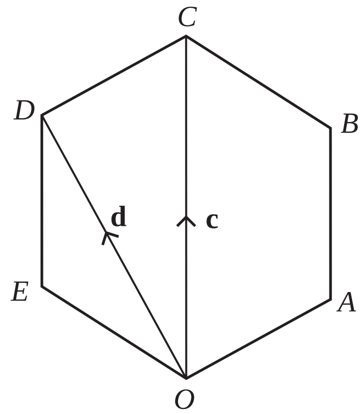
(c) Describe fully a single transformation which maps

(i) triangle *OBA* onto triangle *OQS*, [2]

(ii) triangle *OBA* onto triangle *RDE*, with *O* mapped onto *R* and *B* mapped onto *D*. [2]

(d) **(i)** How many lines of symmetry does the star have? [1]

(ii) When triangle *OQS* is rotated clockwise about *X*, it lies on triangle *PRT*, with *O* on *P*. Write down the angle of rotation. [1]



NOT TO SCALE

OABCDE is a regular hexagon.
With *O* as origin the position vector of *C* is **c** and the position vector of *D* is **d**.

(a) Find, in terms of **c** and **d**,

(i) $\overrightarrow{DC}$, [1]

(ii) $\overrightarrow{OE}$, [2]

(iii) the position vector of *B*. [2]

(b) The sides of the hexagon are each of length 8 cm.

Calculate

(i) the size of angle *ABC*, [1]

(ii) the area of triangle *ABC*, [2]

(iii) the length of the straight line *AC*, [3]

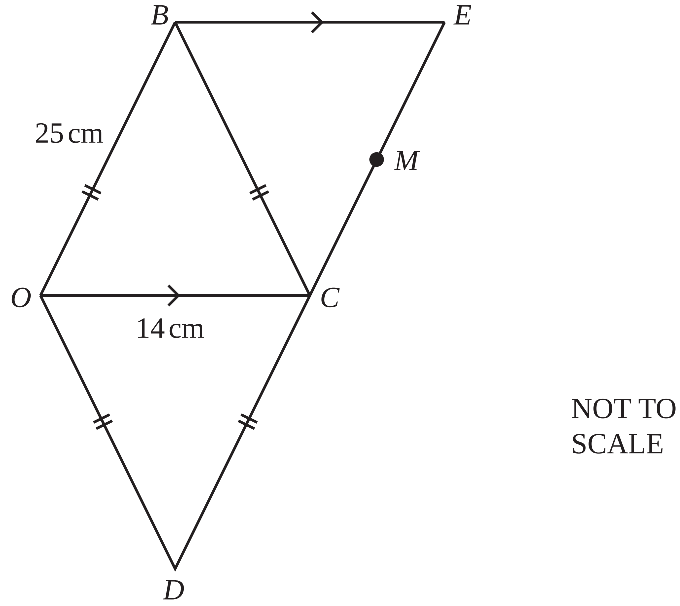
(iv) the area of the hexagon. [3]

(b) $\mathbf{P} = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}$, $\mathbf{Q} = (-1 \quad -2)$, $\mathbf{R} = (1 \quad 2 \quad 3)$, $\mathbf{S} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$.

Only some of the following matrix operations are possible with matrices **P**, **Q**, **R** and **S** above.

PQ, **QP**, **P** + **Q**, **PR**, **RS**

Write down and calculate each matrix operation that is possible. [6]



OBCD is a rhombus with sides of 25 cm. The length of the diagonal *OC* is 14 cm.

(a) Show, **by calculation**, that the length of the diagonal *BD* is 48 cm. [3]

(b) Calculate, correct to the nearest degree,

(i) angle *BCD*, [2]

(ii) angle *OBC*. [1]

(c) $\overrightarrow{DB} = 2\mathbf{p}$ and $\overrightarrow{OC} = 2\mathbf{q}$.
Find, in terms of **p** and **q**,

(i) $\overrightarrow{OB}$, [1]

(ii) $\overrightarrow{OD}$. [1]

(d) *BE* is parallel to *OC* and *DCE* is a straight line.
Find, in its simplest form, $\overrightarrow{OE}$ in terms of **p** and **q**. [2]

(e) *M* is the mid-point of *CE*.
Find, in its simplest form, $\overrightarrow{OM}$ in terms of **p** and **q**. [2]

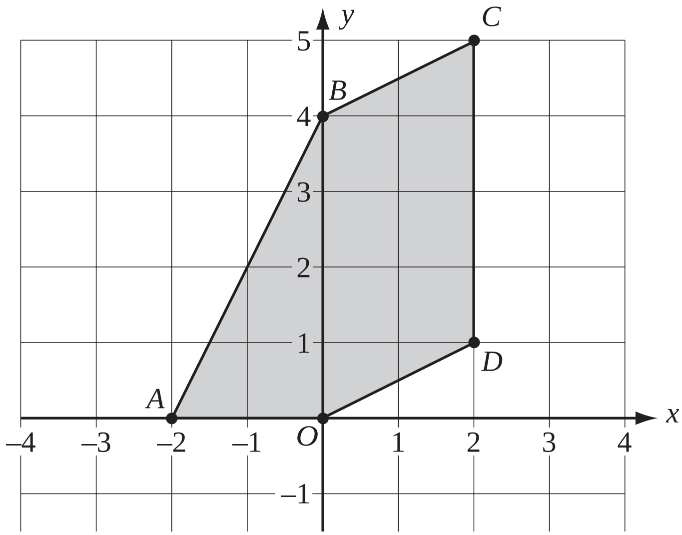
(f) *O* is the origin of a co-ordinate grid. *OC* lies along the *x*-axis and $\mathbf{q} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$.

($\overrightarrow{DB}$ is vertical and $|\overrightarrow{DB}| = 48$.)
Write down as column vectors

(i) **p**, [1]

(ii) $\overrightarrow{BC}$. [2]

6



The pentagon *OABCD* is shown on the grid above.

(a) Write as column vectors

(i) $\overrightarrow{OD}$, [1]

(ii) $\overrightarrow{BC}$. [1]

(b) Describe fully the single transformation which maps the side *BC* onto the side *OD*. [2]

(c) The shaded area inside the pentagon is defined by 5 inequalities.

One of these inequalities is $y \leqslant \frac{1}{2}x + 4$.

Find the other 4 inequalities. [5]

2 **(a)** $\mathbf{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

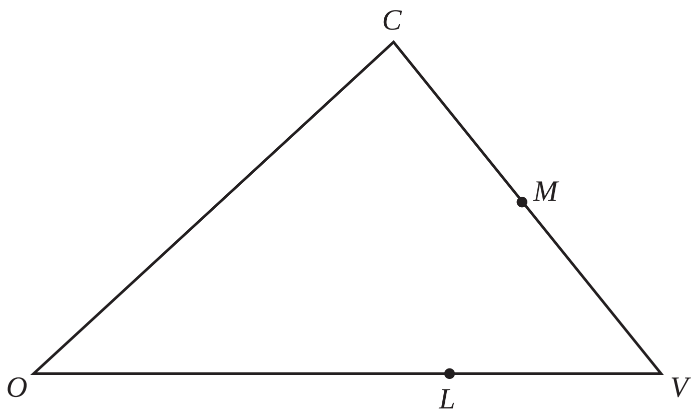
(i) Find, as a single column vector, $\mathbf{p} + 2\mathbf{q}$.

Answer(a)(i) $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]

(ii) Calculate the value of $|\,\mathbf{p} + 2\mathbf{q}\,|$.

Answer(a)(ii) .... [2]

(b)



NOT TO SCALE

In the diagram, $CM = MV$ and $OL = 2LV$.
O is the origin. $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OV} = \mathbf{v}$.

Find, in terms of **c** and **v**, in their simplest forms

(i) $\overrightarrow{CM}$,

Answer(b)(i) .... [2]

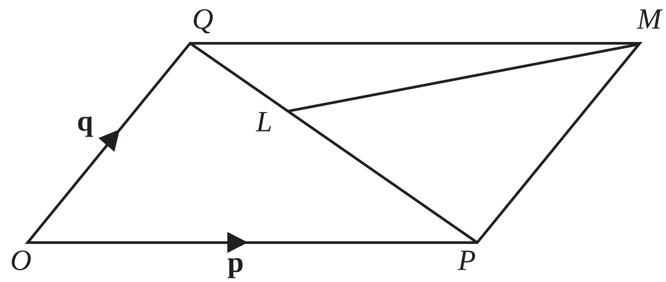
(ii) the position vector of M,

Answer(b)(ii) .... [2]

(iii) $\overrightarrow{ML}$.

6 (a)

NOT TO SCALE



OPMQ is a parallelogram and *O* is the origin.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
L is on *PQ* so that *PL* : *LQ* = 2:1.

Find the following vectors in terms of **p** and **q**. Write your answers in their simplest form.

(i) $\overrightarrow{PQ}$, [1]
(ii) $\overrightarrow{PL}$, [1]
(iii) $\overrightarrow{ML}$, [2]
(iv) the position vector of *L*. [2]

(b) *R* is the point (1,2). It is translated onto the point *S* by the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

(i) Write down the co-ordinates of *S*. [1]
(ii) Write down the vector which translates *S* onto *R*. [1]

(c) The matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents a **single** transformation.

(i) Describe fully this transformation. [3]
(ii) Find the co-ordinates of the image of the point (5, 3) after this transformation. [1]

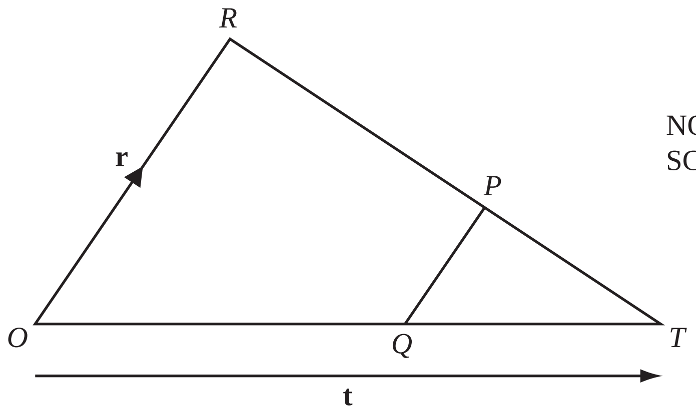
(d) Find the matrix which represents a reflection in the line $y = x$. [2]

(ii) $\overrightarrow{AC} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$.

Work out $\overrightarrow{BC}$ as a column vector.

Answer(b)(ii) $\overrightarrow{BC} = \begin{pmatrix} \\ \\ \end{pmatrix}$ [2]

(c)



NOT TO SCALE

$\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{OT} = \mathbf{t}$.
P is on RT such that $RP : PT = 2 : 1$.
Q is on OT such that $OQ = \frac{2}{3}OT$.

Write the following in terms of **r** and/or **t**.
Simplify your answers where possible.

(i) $\overrightarrow{QT}$

Answer(c)(i) $\overrightarrow{QT}$ = ... [1]

(ii) $\overrightarrow{TP}$

Answer(c)(ii) $\overrightarrow{TP}$ = ... [2]

(iii) $\overrightarrow{QP}$

Answer(c)(iii) $\overrightarrow{QP}$ = ... [2]

(iv) Write down two conclusions you can make about the line segment QP.

Answer(c)(iv) ....

.... [2]

For Examiner's Use

4 **(a)**

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 1 & 2 \end{pmatrix}$$

For Examiner's Use

Find the following matrices.

(i) **AB**

Answer(a)(i) [2]

(ii) **CB**

Answer(a)(ii) [2]

(iii) $\mathbf{A}^{-1}$, the inverse of **A**

Answer(a)(iii) [2]

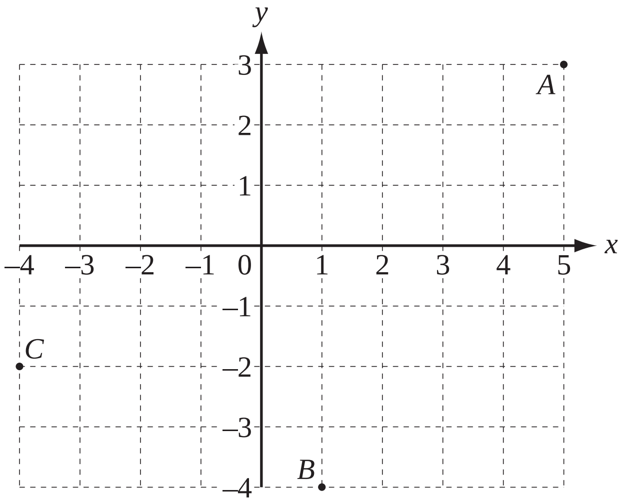
(b) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Answer(b) .... [2]

(c) Find the 2 by 2 matrix that represents an anticlockwise rotation of 90° about the origin.

Answer(c) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

9 **(a)**



The points A (5, 3), B (1, –4) and C (–4, –2) are shown in the diagram.

(i) Write $\overrightarrow{CA}$ as a column vector.

Answer(a)(i) $\overrightarrow{CA} = \begin{pmatrix} \\ \end{pmatrix}$ [1]

(ii) Find $\overrightarrow{CA} - \overrightarrow{CB}$ as a single column vector.

Answer(a)(ii) $\begin{pmatrix} \\ \end{pmatrix}$ [2]

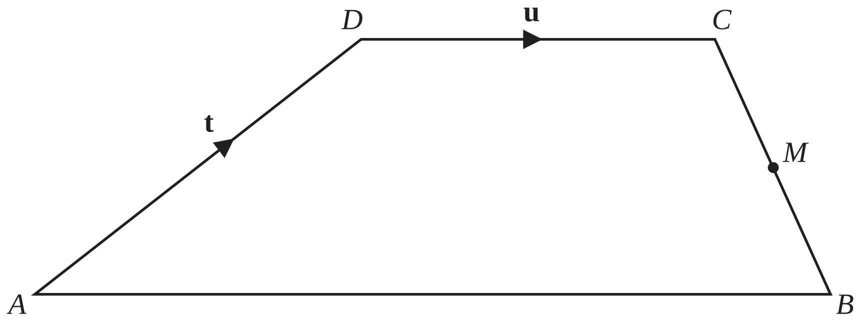
(iii) Complete the following statement.

$\overrightarrow{CA} - \overrightarrow{CB} =$ .... [1]

(iv) Calculate $|\overrightarrow{CA}|$.

Answer(a)(iv) .... [2]

(b)



ABCD is a trapezium with *DC* parallel to *AB* and $DC = \frac{1}{2}AB$.

M is the midpoint of *BC*.

$\overrightarrow{AD} = \mathbf{t}$ and $\overrightarrow{DC} = \mathbf{u}$.

Find the following vectors in terms of **t** and / or **u**.

Give each answer in its simplest form.

(i) $\overrightarrow{AB}$

Answer(b)(i) $\overrightarrow{AB}$ = ... [1]

(ii) $\overrightarrow{BM}$

Answer(b)(ii) $\overrightarrow{BM}$ = ... [2]

(iii) $\overrightarrow{AM}$

Answer(b)(iii) $\overrightarrow{AM}$ = ... [2]

For Examiner's Use

where n is a positive integer and $\binom{\ }{r} = \frac{n!}{(n-r)!r!}$.