10 A toothpaste firm supplies tubes of toothpaste to 5 different stores. The number of tubes of toothpaste supplied per delivery to each store, the sizes and sale prices of the tubes, together with the number of deliveries made to each store over a 3-month period are shown in the table below.

		Number of tubes per delivery			Number of deliveries over 3 months
Size of tube		50 ml	75 ml	100 ml	
Name of store	Alwin	400	300	400	13
	Bestbuy	–	–	600	7
	Costless	400	–	600	10
	Dealwise	500	300	–	5
	Econ	600	600	400	8
Sale price per tube		\$2.10	\$3.00	\$3.75	

(i) Write down two matrices such that the elements of their product under matrix multiplication would give the volume of toothpaste supplied to each store per delivery.

(ii) Write down two matrices such that the elements of their product under matrix multiplication would give the number of tubes of toothpaste of each size supplied by the firm over the 3-month period. Find this product.

(iii) Using the matrix product found in part **(ii)** and a further matrix, find the total amount of money which would be obtained from the sale of all the tubes of toothpaste delivered over the 3-month period.

[8]

1 It is given that $\mathbf{A} = \begin{pmatrix} 5 & 7 \\ 4 & 5 \end{pmatrix}$ and that $\mathbf{A} - 3\mathbf{A}^{-1} - k\mathbf{I} = \mathbf{0}$, where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ is the zero matrix. Evaluate k. [4]

2 Given that $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$, find $\mathbf{B}$ such that $4\mathbf{A}^{-1} + \mathbf{B} = \mathbf{A}^2$. [6]

6 The table below shows

the daily production, in kilograms, of two types, S_1 and S_2, of sweets from a small company,

the percentages of the ingredients A, B and C required to produce S_1 and S_2.

	Percentage			Daily production (kg)
	A	B	C	
Type S_1	60	30	10	300
Type S_2	50	40	10	240

Given that the costs, in dollars per kilogram, of A, B and C are 4, 6 and 8 respectively, use matrix multiplication to calculate the total cost of daily production. [6]

6 It is given that $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 5 & p \end{pmatrix}$ and that $\mathbf{A} + \mathbf{A}^{-1} = k\mathbf{I}$, where p and k are constants and $\mathbf{I}$ is the identity matrix. Evaluate p and k. [6]

1 Given that $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$, find $(\mathbf{A}^2)^{-1}$. [4]

2 A flower show is held over a three-day period – Thursday, Friday and Saturday. The table below shows the entry price per day for an adult and for a child, and the number of adults and children attending on each day.

	Thursday	Friday	Saturday
Price ($) – Adult	12	10	10
Price ($) – Child	5	4	4
Number of adults	300	180	400
Number of children	40	40	150

(i) Write down two matrices such that their product will give the amount of entry money paid on Thursday and hence calculate this product. [2]

(ii) Write down two matrices such that the elements of their product give the amount of entry money paid for each of Friday and Saturday and hence calculate this product. [2]

(iii) Calculate the total amount of entry money paid over the three-day period. [1]

3 **(i)** Given that $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -2 & 5 \end{pmatrix}$, find the inverse of the matrix $\mathbf{A} + \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. [3]

7 Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 8 & 10 \\ -4 & 2 \end{pmatrix}$, find the matrices $\mathbf{X}$ and $\mathbf{Y}$ such that

(i) $\mathbf{X} = \mathbf{A}^2 + 2\mathbf{B}$, [3]

(ii) $\mathbf{YA} = \mathbf{B}$. [4]

4 **(i)** Find, in ascending powers of x, the first 4 terms of the expansion of $(1 + x)^6$. [2]

(ii) Hence find the coefficient of p^3 in the expansion of $(1 + p - p^2)^6$. [3]

5 **(a)** Given that $\mathbf{A} = (2 \quad -4 \quad 1)$ and $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 0 & 5 \\ -2 & 7 \end{pmatrix}$, find the matrix product $\mathbf{AB}$. [2]

(b) Given that $\mathbf{C} = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 6 & -4 \\ 2 & 8 \end{pmatrix}$, find

(i) the inverse matrix $\mathbf{C}^{-1}$, [2]

(ii) the matrix $\mathbf{X}$ such that $\mathbf{CX} = \mathbf{D}$. [2]

5 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} -2 & -1 \\ 6 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ 4 & 3 \end{pmatrix}$. Find matrices **P** and **Q** such that

(i) $\mathbf{P} = \mathbf{B}^2 - 2\mathbf{A}$, [3]

(ii) $\mathbf{Q} = \mathbf{B}(\mathbf{A}^{-1})$. [4]

2 The table shows the number of games played and the results of five teams in a football league.

	Played	Won	Drawn	Lost
Parrots	8	5	3	0
Quails	7	4	1	2
Robins	8	4	0	4
Swallows	7	2	1	4
Terns	8	1	1	6

A win earns 3 points, a draw 1 point and a loss 0 points. Write down two matrices which on multiplication display in their product the total number of points earned by each team and hence calculate these totals. [4]

4 Students take three multiple-choice tests, each with ten questions. A correct answer earns 5 marks. If no answer is given 1 mark is scored. An incorrect answer loses 2 marks. A student's final total mark is the sum of 20% of the mark in test 1, 30% of the mark in test 2 and 50% of the mark in test 3. One student's responses are summarized in the table below.

	Test 1	Test 2	Test 3
Correct answer	7	6	5
No answer	1	3	5
Incorrect answer	2	1	0

Write down three matrices such that matrix multiplication will give this student's final total mark and hence find this total mark. [5]

4 A cycle shop sells three models of racing cycles, *A*, *B* and *C*. The table below shows the numbers of each model sold over a four-week period and the cost of each model in $.

Week \ Model	*A*	*B*	*C*
1	8	12	4
2	7	10	2
3	10	12	0
4	6	8	4
Cost ($)	300	500	800

In the first two weeks the shop banked 30% of all money received, but in the last two weeks the shop only banked 20% of all money received.

(i) Write down three matrices such that matrix multiplication will give the total amount of money banked over the four-week period. [2]

(ii) Hence evaluate this total amount. [4]

9 Given that $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -5 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, calculate

(i) $\mathbf{AB}$, [2]

(ii) $\mathbf{BC}$, [2]

(iii) the matrix $\mathbf{X}$ such that $\mathbf{AX} = \mathbf{B}$. [4]

2 Given that $\mathbf{A} = \begin{pmatrix} 7 & 6 \\ 3 & 4 \end{pmatrix}$, find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$7x + 6y = 17,$$
$$3x + 4y = 3.$$

[4]

8 Given that $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 0 \\ 1 & 4 \end{pmatrix}$, find

(i) $3\mathbf{A} - 2\mathbf{B}$, [2]

(ii) $\mathbf{A}^{-1}$, [2]

(iii) the matrix $\mathbf{X}$ such that $\mathbf{XB}^{-1} = \mathbf{A}$. [3]

8 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 & 7 \\ 1 & -5 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 8 & 6 \end{pmatrix}$, calculate

(i) $2\mathbf{A}$, [1]

(ii) $\mathbf{B}^2$, [2]

(iii) $\mathbf{BA}$. [2]

(b) **(i)** Given that $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 7 & 6 \end{pmatrix}$, find $\mathbf{C}^{-1}$. [2]

(ii) Given also that $\mathbf{D} = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$, find the matrix $\mathbf{X}$ such that $\mathbf{XC} = \mathbf{D}$. [2]

1 Given that $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, find the value of each of the constants m and n for which

$$\mathbf{A}^2 + m\mathbf{A} = n\mathbf{I},$$

where $\mathbf{I}$ is the identity matrix. [4]

8 Given that $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$, use the inverse matrix of $\mathbf{A}$ to

(i) solve the simultaneous equations

$$y - 4x + 8 = 0,$$

$$2y - 3x + 1 = 0,$$

(ii) find the matrix $\mathbf{B}$ such that $\mathbf{BA} = \begin{pmatrix} -2 & 3 \\ 9 & -1 \end{pmatrix}$.

[8]

5 A large airline has a fleet of aircraft consisting of 5 aircraft of type A, 8 of type B, 4 of type C and 10 of type D. The aircraft have 3 classes of seat known as Economy, Business and First. The table below shows the number of these seats in each of the 4 types of aircraft.

Class of seat / Type of aircraft	Economy	Business	First
A	300	60	40
B	150	50	20
C	120	40	0
D	100	0	0

(i) Write down two matrices whose product shows the total number of seats in each class.

(ii) Evaluate this product of matrices.

On a particular day, each aircraft made one flight. 5% of the Economy seats were empty, 10% of the Business seats were empty and 20% of the First seats were empty.

(iii) Write down a matrix whose product with the matrix found in part **(ii)** will give the total number of empty seats on that day.

(iv) Evaluate this total.

[6]

8 **(a)** The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 2 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 1 & 3 & 4 \\ 1 & 5 & 6 & 7 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 9 \\ 10 \end{pmatrix}$. Write down, but do not evaluate, matrix products which may be calculated from the matrices **A**, **B** and **C**. [2]

(b) Given that $\mathbf{X} = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} 2x & 3y \\ x & 4y \end{pmatrix}$, find the value of x and of y such that

$$\mathbf{X}^{-1}\mathbf{Y} = \begin{pmatrix} -12x + 3y & 6 \\ -7x + 3y & 6 \end{pmatrix}.$$

[6]

8 **(a)** Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 & 7 \\ 1 & -5 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 8 & 6 \end{pmatrix}$, calculate

(i) $2\mathbf{A}$, [1]

(ii) $\mathbf{B}^2$, [2]

(iii) $\mathbf{BA}$. [2]

(b) **(i)** Given that $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 7 & 6 \end{pmatrix}$, find $\mathbf{C}^{-1}$. [2]

(ii) Given also that $\mathbf{D} = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$, find the matrix $\mathbf{X}$ such that $\mathbf{XC} = \mathbf{D}$. [2]

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(iii) Calculate the total amount of entry money paid over the three-day period. [1]

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