11 Express $2x^2 - 8x + 5$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 8x + 5$ for the domain $0 \leqslant x \leqslant 5$.

(i) Find the range of f.

(ii) Explain why f does not have an inverse.

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 8x + 5$ for the domain $x \geqslant k$.

(iii) Find the smallest value of k for which g has an inverse.

(iv) For this value of k, find an expression for g^{-1}.

[10]

2 **(i)** Sketch, on the same diagram, the graphs of $y = |x| + 1$ and $y = |2x - 3|$.

(ii) State the number of solutions of the equation $|2x - 3| = |x| + 1$.

[4]

11 Functions f and g are defined, for $x \in \mathbb{R}$, by

$$\mathrm{f} : x \mapsto 3x - 7,$$

$$\mathrm{g} : x \mapsto \frac{12}{x - 2}, \quad x \neq 2.$$

(i) Find f^{-1} and g^{-1} in terms of x, stating the value of x for which g^{-1} is not defined. [3]

(ii) Find the values of x for which $\mathrm{fg}(x) = x$. [3]

(iii) Sketch the graphs of f and f^{-1} on the same diagram, giving the coordinates of the points of intersection of each graph with the axes. [3]

3 **(i)** Sketch on the same diagram the graphs of $y = |2x + 3|$ and $y = 1 - x$. [3]

(ii) Find the values of x for which $x + |2x + 3| = 1$. [3]

6 **(a)** Find the values of x for which $2x^2 > 3x + 14$. [3]

(b) Find the values of k for which the line $y + kx = 8$ is a tangent to the curve $x^2 + 4y = 20$. [3]

7 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \mathrm{e}^x,$$

$$\mathrm{g} : x \mapsto 2x - 3.$$

(i) Solve the equation $\mathrm{fg}(x) = 7$. [2]

Function h is defined as gf.

(ii) Express h in terms of x and state its range. [2]

(iii) Express h^{-1} in terms of x. [2]

9 Express $6 + 4x - x^2$ in the form $a - (x + b)^2$, where a and b are integers. [2]

(i) Find the coordinates of the turning point of the curve $y = 6 + 4x - x^2$ and determine the nature of this turning point. [3]

The function f is defined by $f : x \mapsto 6 + 4x - x^2$ for the domain $0 \leqslant x \leqslant 5$.

(ii) Find the range of f. [2]

(iii) State, giving a reason, whether or not f has an inverse. [1]

6 Given that each of the following functions is defined for the domain $-2 \leqslant x \leqslant 3$, find the range of

(i) $f : x \mapsto 2 - 3x$, [1]

(ii) $g : x \mapsto |2 - 3x|$, [2]

(iii) $h : x \mapsto 2 - |3x|$. [2]

State which of the functions f, g and h has an inverse. [2]

9 **(i)** Determine the set of values of k for which the equation

$$x^2 + 2x + k = 3kx - 1$$

has no real roots. [5]

(ii) Hence state, giving a reason, what can be deduced about the curve $y = (x + 1)^2$ and the line $y = 3x - 1$. [2]

8 The equation of the curve C is $2y = x^2 + 4$. The equation of the line L is $y = 3x - k$, where k is an integer.

(i) Find the largest value of the integer k for which L intersects C. [4]

(ii) In the case where $k = -2$, show that the line joining the points of intersection of L and C is bisected by the line $y = 2x + 5$. [4]

10 The functions f and g are defined, for $x \in \mathbb{R}$, by

$f : x \mapsto 3x - 2$,

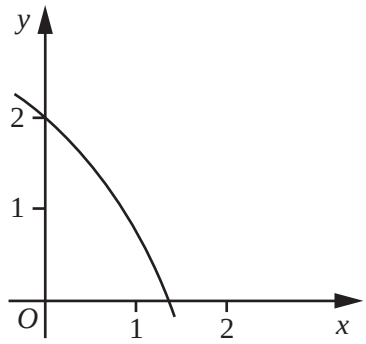
$g : x \mapsto \dfrac{7x - a}{x + 1}$, where $x \neq -1$ and a is a positive constant.

(i) Obtain expressions for f^{-1} and g^{-1}. [3]

(ii) Determine the value of a for which $f^{-1}g(4) = 2$. [3]

(iii) If $a = 9$, show that there is only one value of x for which $g(x) = g^{-1}(x)$. [3]

4



The diagram shows part of the curve $y = f(x)$, where $f(x) = p - e^x$ and p is a constant. The curve crosses the y-axis at $(0, 2)$.

(i) Find the value of p. [2]

(ii) Find the coordinates of the point where the curve crosses the x-axis. [2]

(iii) Copy the diagram above and on it sketch the graph of $y = f^{-1}(x)$. [2]

1 A triangle has a base of length $(13 - 2x)$ m and a perpendicular height of x m. Calculate the range of values of x for which the area of the triangle is greater than $3\,m^2$. [3]

(i) Express $2x^2 - 8x + 3$ in the form $a(x + b)^2 + c$, where a, b and c are integers. [2]

A function f is defined by $\text{f} : x \mapsto 2x^2 - 8x + 3, \quad x \in \mathbb{R}$.

(ii) Find the coordinates of the stationary point on the graph of $y = \text{f}(x)$. [2]

(iii) Find the value of $\text{f}^2(0)$. [2]

A function g is defined by $\text{g} : x \mapsto 2x^2 - 8x + 3, \quad x \in \mathbb{R}$, where $x \leqslant N$.

(iv) State the greatest value of N for which g has an inverse. [1]

(v) Using the result obtained in part **(i)**, find an expression for g^{-1}. [3]

OR

The equation of a curve is $y = 10 - x^2 + 6x$.

(i) Find the set of values of x for which $y \geqslant 15$. [3]

(ii) Express y in the form $a - (x + b)^2$, where a and b are integers. [2]

(iii) Hence, or otherwise, find the coordinates of the stationary point on the curve. [2]

Functions f and g are defined, for $x \in \mathbb{R}$, by

$\text{f} : x \mapsto 10 - x^2 + 6x$,

$\text{g} : x \mapsto 2x - k$, where k is a constant.

(iv) Find the value of k for which the equation $\text{gf}(x) = 0$ has two equal roots. [3]

4 **(i)** Sketch, on the same diagram, the graphs of $y = x - 3$ and $y = |2x - 9|$. [3]

(ii) Solve the equation $|2x - 9| = x - 3$. [2]

3 Sketch the graph of $y = |x^2 - 8x + 12|$. [4]

6 The function f is defined by $f(x) = 2 + \sqrt{x - 3}$ for $x \geqslant 3$. Find

(i) the range of f, [1]

(ii) an expression for $f^{-1}(x)$. [2]

The function g is defined by $g(x) = \frac{12}{x} + 2$ for $x > 0$. Find

(iii) gf(12). [2]

7 The function f is defined by

$$f(x) = (2x + 1)^2 - 3 \quad \text{for } x \geqslant -\frac{1}{2}.$$

Find

(i) the range of f, [1]

(ii) an expression for $f^{-1}(x)$. [3]

The function g is defined by

$$g(x) = \frac{3}{1 + x} \quad \text{for } x > -1.$$

(iii) Find the value of x for which $fg(x) = 13$. [4]

11 The function f is defined by

$$f(x) = (x + 1)^2 + 2 \text{ for } x \geqslant -1.$$

Find

(i) the range of f, [1]

(ii) $f^2(1)$, [1]

(iii) an expression for $f^{-1}(x)$. [3]

The function g is defined by

$$g(x) = \frac{20}{x + 1} \text{ for } x \geqslant 0.$$

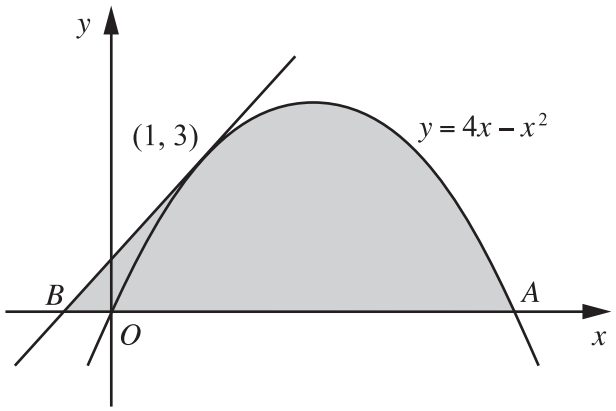
Find

(iv) $g^{-1}(2)$, [2]

(v) the value of x for which $fg(x) = 38$. [4]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows the curve $y = 4x - x^2$, which crosses the x-axis at the origin O and the point A. The tangent to the curve at the point (1, 3) crosses the x-axis at the point B.

(i) Find the coordinates of A and of B. [5]

(ii) Find the area of the shaded region. [5]

OR

Solutions to this question by accurate drawing will not be accepted.

The points $A(-2, 2)$, $B(4, 4)$ and $C(5, 2)$ are the vertices of a triangle. The perpendicular bisector of AB and the line through A parallel to BC intersect at the point D. Find the area of the quadrilateral $ABCD$. [10]

It is given that $f(x) = 4x^2 + kx + k$.

(i) Find the set of values of k for which the equation $f(x) = 3$ has no real roots. [5]

In the case where $k = 10$,

(ii) express $f(x)$ in the form $(ax + b)^2 + c$, [3]

(iii) find the least value of $f(x)$ and the value of x for which this least value occurs. [2]

The functions f, g and h are defined, for $x \in \mathbb{R}$, by

$$f(x) = x^2 + 1,$$

$$g(x) = 2x - 5,$$

$$h(x) = 2^x.$$

(i) Write down the range of f. [1]

(ii) Find the value of $gf(3)$. [2]

(iii) Solve the equation $fg(x) = g^{-1}(15)$. [5]

(iv) On the same axes, sketch the graph of $y = h(x)$ and the graph of the inverse function $y = h^{-1}(x)$, indicating clearly which graph represents h and which graph represents h^{-1}. [2]

7 **(i)** Sketch the graph of $y = |3x + 9|$ for $-5 < x < 2$, showing the coordinates of the points where the graph meets the axes. [3]

(ii) On the same diagram, sketch the graph of $y = x + 6$. [1]

(iii) Solve the equation $|3x + 9| = x + 6$. [3]